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Market procyclicality and systemic risk

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We develop a model that captures, at the same time, the temporal dynamics of single-firm credit risk and the contagion across banks via a network of obligations and common assets. In particular, we enrich the continuous-time modelling approach of default by accounting explicitly for the procyclical loop between asset prices and leverage. Contagion can spread well before any default occurs, through the value of the obligations held by counterparties. Moreover, the extent of procyclicality effects depends explicitly on the structure of both the interbank network and the asset bank network. We analyse the model in a simplified scenario of a densely connected core of banks and we carry out a systematic investigation of how procyclicality emerges from the multiplicative interplay of market illiquidity and tightness of capital requirements.

Keywords: Financial networks; Continuous-time; Systemic risk; Procyclicality; Leverage; Market liquidity

JEL Classification: G20, G28

1. Introduction

In a mark-to-market financial system, asset price movements have an instantaneous impact on the net worth of the market participants who hold those assets. This effect is amplified when financial institutions use borrowing to leverage their exposure to risky assets. Moreover, there are also indirect spillover effects on market participants who hold claims on other participants holding those assets. Therefore, aggregate asset-price shocks can propagate through the financial system via both direct and indirect linkages. A recent body of research has investigated the relation between price changes and subsequent balance sheet adjustments resulting from the fact that institutions actively manage their value-at-risk (Danielsson and Zigrand 2008, Adrian and Shin 2010, 2011b, Danielsson et al. 2012). This practice leads to spirals of asset devaluation (or overvaluation). Accordingly, a debate has developed regarding the appropriate policy instruments to mitigate the procyclical effects arising from the interplay between leverage and mark-to-market asset valuation (see e.g. BIS 2010, EC 2011, Arnold et al. 2012).

Our goal is to advance our understanding of procyclical effects on systemic risk§ by combining, for the first time, tools coming from two strands of literature that have remained disconnected so far. On the one hand, the vast literature that builds on Merton (1974), Black and Cox (1976) allows the valuation of firms’ obligations through the estimation of their time to default and it has provided practitioners with many tools to manage credit risk. However, the interaction among firms through mutual interbank obligations, which is especially relevant in the interbank market, has been neglected.

On the other hand, a different stream of works, pioneered by Eisenberg and Noe (2001) investigate the notion of default probability in a system context, i.e. when banks are connected in a network of liabilities. This literature considers the liquidation value of corporate debts only at the time of default (e.g. Cifuentes et al. 2005, Elsinger et al. 2006). Indeed, the value of interbank claims depends on the solvency of the counterparties at the maturity of the contracts and it is determined as the fixed point of a so-called `fictitious sequential default’ algorithm. Then, starting from a given exogenous shock on one or more banks, one can measure ex-post the impact of the shock to the system and investigate, for instance, which structure are more resilient to systemic risk (Battiston et al. 2012a, Roukny et al. 2013). However, this approach fails to describe how default probability evolves over time before the shocks are realized and before the maturity of the claims. This is a limitation especially when the asset price dynamics is not time-homogeneous.

Our objective here is to capture at the same time the dynamics of single-firm credit risks as well as their interaction. Our effort in combining the two strands of literature is grounded on the fact that both use the structural approach, which is based on the economic fundamentals of the balance sheet structure of the firm.

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§‘Systemic risk’ is meant here as the risk of a systemic default and in the following it will be used interchangeably with the term ‘probability of systemic default’.

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The contributions of our paper are the following. First, we reformulate the dynamics of procyclicality (that other works have modelled from a macroeconomic perspective) in a continuous-time setting with n interacting banks. Since most of the practitioners’ reference models deal with continuous time processes for an isolated bank, our framework helps bridging the conversation on macroprudential policies with the literature in mathematical finance. Second, our model provides a dynamic description of the mechanism of contagion via both common assets and liability interlocks. Contagion spreads, well before any default occurs, through the value of the obligations held by counterparties. In particular, in the model, the procyclicity from asset prices to balance sheets depends explicitly on the structure of both the interbank network and the asset bank network. Third, thanks to some simplifications, we were able to carry out a systematic investigation of how procyclicity emerges from the interplay of market illiquidity and tightness of capital requirements. We find that the extent of the procyclicity can either be contained by acting on the timing of the capital requirements and/or on the asset market liquidity (because the two factors interact multiplicatively).

In detail more, we model a financial system that is composed of leveraged financial institutions (hereafter, ‘banks’) that manage diversified but possibly overlapping portfolios of real-economy-related assets (hereafter, ‘external assets’). Banks hold claims on each other (hereafter, ‘interbank claims’), the value of which depend on the obligor’s leverage. Banks also borrow funds from market players outside the banking network (hereafter, ‘external funds’). Further, we model dynamic balance-sheet management. As noted by Adrian and Shin (2010), the practice of adjusting the value-at-risk (VaR) to a given target level is equivalent to maintaining leverage close to a constant value of financial reporting leverage (hereafter, ‘target leverage’). The paper formalizes an accounting rule in support of Adrian and Shin (2010) findings, whereby banks sell or buy external assets in response to price movements. An increase (decrease) in prices induces an expansion (contraction) of the balance sheets, which generates further price movements in the same direction. Consequently, prices exhibit a stochastic dynamics whose return is influenced by bank trades. In other words, the combination of (1) balance-sheet management and (2) the price response generates a positive feedback loop between leverage and prices (i.e. leverage-price cycle) that may amplify the effects of common shocks into a spiral of asset price devaluation or overvaluation.

We thus introduce a table of market procyclicity in the exogenous parameters ε and γ. The parameter ε represents the level of bank compliance with capital requirements, i.e. promptness in adjusting leverage deviations from the target level. Values of ε that are close to zero denote banks’ sluggish adjustments, whereas values of ε that are close to one mean that banks react very promptly. In other words, ε can be understood as a scaling factor of the trading size engendered by the accounting rule. The parameter γ captures the market impact (i.e. the average price response to bank trades). γ is also related to the market liquidity risk. We define a liquid market as a market where participants can execute large transactions at short notice with minimal impact on the price. An asset will be liquid if it is traded in a liquid market. Therefore, low (high) values of γ imply a liquid (illiquid) market. Thus, the market is said to be weakly (strongly) procyclical when both ε and γ are small (large).

Although the model can accommodate various market microstructures and heterogeneous players, in the analysis we focus on a scenario that includes a homogeneous and tightly-knit financial system. In this context, we test the resilience of the system by introducing a common market-wide price shock (hereafter, ‘price shock’). The price shock is generic because we do not distinguish between fundamental and nonfundamental shocks. In addition, the asset market is incomplete in the sense that banks cannot insure themselves against the price shock (i.e. there are no Arrow securities that are contingent on the shock).

Our findings are as follows. A negative shock to asset prices depletes capital and increases leverage. When ε is not too small and γ is sufficiently high, (i.e. when the price response to bank trades is more rapid than the adjustment in leverage), banks continue pursuing their target leverage without reaching it. In so doing, banks generally amplify the effect of the initial price shock. Therefore, the probability of systemic default is higher. Conversely, when ε is high and γ is very close to zero (i.e. when the market is so liquid that the price response to bank trades is considerably weaker than the adjustments in leverage), banks manage to promptly track their target leverage without triggering an amplification of the initial price shock. In this case, the probability of systemic default remains low. In addition, the mix of funding sources is not irrelevant: the probability of systemic default increases with the average bank exposures to external funds relative to interbank funds.

Overall, this paper sheds light on the mechanisms that govern the emergence of the tension between (1) the individual incentive to target a given VaR (relative to its economic capital) and (2) systemic risk. In terms of their policy implications, our results are significant because they suggest that in the presence of price shocks, counter-cyclical policies should take into account the interplay between banks’ procyclical behaviour and the time-variation of asset market liquidity.

This paper is structured as follows. We begin with a review of the related literature. Then, section 2 introduces the model. In section 3, we simplify the modelling framework using mean-field approximation and conduct a numerical analysis of the probability of systemic default. Section 4 presents the results. Section 5 concludes and considers the policy implications of the results.

1.1. Related literature

This paper is related to the following discussions in the literature. Several authors have investigated financial contagion in the interbank market (see e.g. Freixas et al. 2000, Allen and Gale 2001, Furfine 2003, Elsinger et al. 2006, Tasca and Battiston 2011, Battiston et al. 2012b). Other studies have investigated contagion effects and systemic risk mediated by common asset holdings (see e.g. Kiyotaki and Moore 2002, Allen et al. 2012, Tasca et al. 2014). As our paper adopts a balance-sheet approach, the results are pertinent to the literature on the amplification of financial shocks via balance
sheet transmission mechanisms (Kashyap and Stein 2000, Brunnermeier 2008)—a literature review on this topic is carried out in Krishnamurthy (2010). Moreover, the paper offers a first attempt to link the balance-sheet approach to a dynamic stochastic setting in a network context. Using this framework, we model the procyclicality of leverage, the empirical evidence of which has been documented mainly in Adrian and Shin (2010, 2008a) and Greenlaw et al. (2008). In particular, we study the macro-level consequences of procyclical leverage and analyse the mutual influence between the contraction and expansion of banks’ balance sheets and asset prices. In a different setting, Adrian and Shin (2011b) offer a two-stage principal-agent model for use in studying the relationship between balance sheet leverage and the riskiness of bank assets. Instead, our network approach is more related to the recent literature on the network-based ‘leverage accelerator’ mechanism (see e.g. Bargigli et al. 2014). Moreover, as liquidity risk is embedded in our model, the paper complements a growing literature on funding and market liquidity problems that in the recent US crisis (2007–2009) started the systemic liquidity spirals (see e.g. Garleanu and Pedersen 2007).

Our model partially resists to the assumptions of ‘equity rationing’ (see e.g. Stiglitz and Weiss 1981) and ‘debt overhang’, the gist that, under certain conditions, it is difficult or inconvenient for banks to raise new equity; evidence and possible causes are discussed in Adrian and Shin (2011a). Moreover, the idea that capital scarcity may lead to a ‘credit crunch’ has been investigated by Bernanke et al. (1991), Calomiris and Wilson (1998), among others. Finally, the problem of the procyclical effects of capital adequacy regulation is a classic one (Merton 1974, Black and Cox 1976), which has provided many credit-risk management tools.

However, as mentioned above, that first passage time problem, this is in line with the past literature on Merton (1974) and Black and Cox (1976), which has provided practitioners with many credit-risk management tools.

2. Model

2.1. The interbank network

In the following, we describe a system of banks with balance sheets interconnected directly via interbank liabilities and interconnected indirectly via common exposures to external assets. The sources of funding also comprise external liabilities, so that the system is not closed. Our first goal will be to characterize how the leverage of each bank depends on the leverage of the others via direct and indirect connections.

Let time be indexed as \( t \in [0, T] \), and consider an interbank market composed of a set \( \Omega_n \) of \( n \) risk-averse banks bound by market risk-based capital requirements. To keep the notation simple, whenever it is unambiguous given the context that a variable is time-dependent, we will omit the index \( t \). The balance-sheet identity of bank \( i \in \Omega_n \) is given by

\[
a_i = h_i + b_i + e_i,
\]

where \( a_i \) is the market value of bank \( i \)'s assets, \( h_i \) is the book value of bank \( i \)'s obligations to other banks in the network (i.e. zero-coupon bonds),

\[ b_i \]

is the book value of bank \( i \)'s external funds with actors outside the banking system and \( e_i \) is the equity value.

It is assumed that each bank invests in two asset classes composed of: (i) \( n-1 \) obligations, each issued by one of the other banks in the system and (ii) \( m \) homogeneous investment opportunities that belong to the set \( \Omega_m \) of external assets related to the real side of the economy. Hereafter, for the sake of simplicity, we omit the lower and upper bounds of the summations. It remains understood that, in the summation for external assets, the index ranges from 1 to \( m \), and that, in the summation for banks, the index ranges from 1 to \( n \). The asset side in equation (1) reads

\[
a_i = \sum_i Q_{ij}s_j + \sum_j W_{ij}h_j.
\]

In the equation above, \( Q_{ij}(\geq 0) \) is the quantity of the external asset \( j \) held by \( i \); \( s_j \) is the price of the external assets \( j \) (with \( W_{ij} = 0 \)) and \( h_j \) is the quantity of debt issued by \( i \) and held by \( j \). Finally, \( h_j \) is the present market value of bank \( j \)'s debt. Since we assume the credit contracts among banks imply only one payment at the maturity (i.e. zero-coupon bonds with a single cash-flow frequency at maturity), the expression of the discounted value of the bond is:

\[
h_i := h_i / [1 + r_i]^{T-t},
\]

where \( r_i \) is the rate of return, \( (T-t) \) the time to maturity and \( c_i = r_i - r_f \) the credit spread (premium), over the risk free rate \( r_f \), paid by the bank to the bond holders.

Symmetrically, the bank’s fundings comprise liabilities to the interbank market and external funds, which are assumed to be available at fixed cost without limitation. The rationale for this assumption is that this work aims at studying only the leverage-price cycle. Further effects such as that one on the interest rate applied to external funds would confound.
our results. However, it is reasonable to expect that it would reinforce the effect of procyclicality.

Correspondingly, bank \( i \)'s balance sheet can be represented as follows.

\[
\begin{array}{l|l}
\text{Assets} & \text{Liabilities} \\
\sum_j Q_{i,j} l_j & h_j \\
\sum_j W_{i,j} h_j & b_j \\
\end{array}
\]

Figure 1 illustrates one possible architecture of the financial system described in our model. In general, there can be overlaps among banks’ portfolios of external assets, external funds and interbank claims. For instance, banks 1 and 2 have one external asset in common, and both hold claims against bank 4. Bank 4 shares the same external fund with bank 3.

2.2. Leverage and default events

A convenient way to look at the problem of default is to analyse the evolution of the ratio \( \phi_i \) of debt to asset:

\[
\phi_i := \frac{h_i + b_i}{a_i}, \quad \text{with} \quad \begin{cases} 
1 & \text{default boundary} \\
\epsilon \to 0^+ & \text{safe boundary}.
\end{cases}
\]

The literature on balance sheet interlock building on Eisenberg and Noe (2001) usually assumes that default occurs when the bank’s equity goes to zero during the process of default cascade (see e.g. Gai and Kapadia 2010, Battiston et al. 2012a). Similarly, here we are interested in the time \( t \leq T \) at which bank’s equity goes to zero or below. The reason is that we have in mind the following scenario. The external funding on the liability side of the bank are short term compared to the interbank debt. If the bank’s equity goes to zero, short-term creditor believe it will default on its interbank debt at time \( T \) and do not roll-over their loans to the bank even if the bank has not yet really defaulted on interbank debt. This implies that practically the bank is in default already at time \( t \).

Because equity is zero or negative if and only if the ratio of debt to asset is equal to one are above, we define the exit time as:

\[
\tau = \inf \{ t > 0 | \phi(t) \geq 1 \},
\]

and the default probability as:

\[
P_t = P\{ \tau_t < T \},
\]

where \( T_i \) is the maturity of bank \( i \)'s interbank loan.

2.3. Leverage in system context

Combining together equations (2)–(4), we obtain:

\[
\phi_i = (h_i + b_i) / \left( \sum_j Q_{i,j} l_j + \sum_j W_{i,j} h_j \right)
\]

\[
(7a)
\]

\[
= (h_i + b_i) / \left( \sum_j Q_{i,j} l_j + \sum_j W_{i,j} h_j / \left[ 1 + r_j \right]^{T-t} \right).
\]

\[
(7b)
\]

Theoretical and empirical evidence have shown that there are multiple control variables affecting the credit spread, such as the firm’s leverage, the volatility of the underlying assets or the liquidity risk (Collin-Dufresne et al. 2001).

The rational adopted in our paper is the following. It is known that the main determinant of the credit spread is the distance to default, which, in turn, depends on how far the leverage \( \phi_i \) is from the default boundary. The credit spread must thus be function \( f \) of the leverage: \( c_i = f(\phi_i) \). Unfortunately, because of the endogenous asset price dynamics adopted in our model (see section 2.5), \( f \) is likely to be highly non-linear and its closed form solution cannot be derived. However, we can expect \( f \) to be a non-decreasing function of \( \phi_i \): indeed, the closer \( \phi_i \) to one, the higher the probability of default and thus the higher the credit spread \( c_i \). In this paper, for the sake of simplicity, we take a parsimonious first-order linear approximation, as follows,

\[
\phi_i = \beta \phi_i,
\]

where the parameter \( \beta > 0 \) and can be understood as the responsiveness of the rate of return to the leverage. This assumption captures the main basic feature that we expect from a credit spread (namely, to increase with the default probability) and allows at the same time to simplify the analysis later on.†

Then, by replacing equation (8) into equation (3) we have:

\[
b_i := h_i / \left[ 1 + r_f + \beta \phi_i \right]
\]

where w.l.g. \( T - t = 1 \). This means that banks issue 1-year maturity obligations that are continuously rolled over. Notice that equation (9) implies that even in the case of a high default probability, bank debts are still priced at a positive market value. Namely, for \( \phi_i \to 1 \), \( h_i \to h_i / \left[ 1 + r_f + \beta \right] > 0 \). This means that, creditors are assumed to partially recover their credits in case of default. The recovery rate can be implicitly determined as shown in Tasca and Battiston (2011). Now, by using equation (9) we can rewrite equation (7b) as:

\[
\phi_i = (h_i + b_i) / \left( \sum_j Q_{i,j} l_j + \sum_j W_{i,j} h_j / \left[ 1 + r_f + \beta \phi_j \right]^{T-t} \right).
\]

Equation (10) highlights a non-linear dependence of \( \phi_i \) from the leverage \( \phi_j \) of the other banks to whom \( i \) is exposed via the matrix \( W \). Recent works based on the ‘clearing payment vector’ mechanism (e.g. Cifuentes et al. 2005) provide a ‘fictitious sequential default’ algorithm to determine the liquidation equilibrium value of interbank claims at their date of maturity. In this respect, equation (9) together with equation (10) capture, even before the maturity of the debts, the market value of interbank claims in the building up of the distress spreading from one bank to another. This is a forward looking measure of systemic risk that estimates at any point in time the likelihood of a system collapse and combines into a single figure asset values, business risk and leverage.

†Notice that in our setting, borrowing existing formulas from other continuous-time models such as the Merton formula to characterize the credit spread, would lead to some problems. Indeed, the Merton’s formula assumes that the underlying firm value follows a geometric Brownian motions with constant drift and variance (while here they are endogenously time varying) and it further ignores any interbank interaction (which here plays an important role).
2.4. Target leverage. An accounting rule

The VaR is widely considered by financial institutions as part of their risk management procedure. As in Shin (2008), we assume that each bank \( i \in \Omega \) adjusts its balance sheet to maintain a market equity \( e_i \) equal to its VaR. In other words, the VaR is the equity capital that a bank must hold to remain solvent with probability \( c \). As explained below, this practice is equivalent to targeting a reference leverage \( \phi^*_i \). Let us use \( V_i \) to denote the VaR per dollar of assets held by \( i \). If bank \( i \) maintains equity capital \( e_i \) to meet total VaR, then we have \( e_i = V_i \times a_i \Rightarrow a_i - h_i + b_i = V_i \times a_i \Rightarrow h_i + b_i = a_i \times (1 - V_i) \). This implies that \( i \) has a target leverage \( \phi^*_i = (h_i + b_i)/a_i = (1 - V_i) \).

The accounting rule that allows bank \( i \) to keep its leverage close to \( \phi^*_i \) over time finds its economic ground in Adrian and Shin (2010), Adrian and Shin (2008a) and operates as follows. In a falling market (i.e. under slumping external asset prices), the leverage decreases (i.e. \( \phi_i > \phi^*_i \)). In this scenario, the bank shrinks its balance sheet by selling (a portion of) its external assets and paying back (a portion of) its external debts with the proceeds. In a rising market (i.e. under booming external asset prices), the leverage decreases (i.e. \( \phi_i < \phi^*_i \)). In such a scenario, the bank expands its balance sheet by taking on additional external funds to invest the ‘fresh’ cash in external activities. Therefore, the dynamics of banks’ balance sheets may reinforce cyclical upturns and downturns. To derive a formal accounting rule, we make the following assumptions:

1. Bank \( i \) can neither issue new equity or replace equity with debts.†
2. The notional value of bank \( i \)’s interbank debts is constant over time, i.e. \( h_i(t) = h_i \) for all \( t \geq 0 \).‡

Notice that the second assumption does not prevent \( i \) from changing counterparties or adjusting their relative shares in \( i \)’s interbank debts. Moreover, it does not prevent the market value of the interbank claims from changing over time based on the time-varying credit worthiness of the debt issuer (see equation (9)). The two assumptions above, together with the balance sheet identity in equation (1), imply that any variation in the external assets held by bank \( i \) will correspond to an equal amount of variation in the external funds on the liability side.

Formally, under the accounting constraints \( Q_i = \sum_l Q_{il} \); \( dQ_i \geq -Q_i \); \( db_i \geq -b_i \), we obtain the following accounting rule (whose derivation is provided in appendix 1).

\[
\frac{db_i}{b_i} = \left( \frac{\varepsilon_i}{\kappa_i} \right) \left( \phi^*_i - \phi_i \right) \left( 1 - \frac{\phi^*_i}{\phi_i} \right). \tag{11}
\]

\[
\frac{dQ_{il}}{Q_{il}} = \left( \frac{\varepsilon_i}{\alpha_{il}} \right) \left( \phi^*_i - \phi_i \right) \left( 1 - \frac{\phi^*_i}{\phi_i} \right). \tag{12}
\]

The parameter \( \varepsilon_i \in (0, 1] \) measures the promptness of \( i \) in pursuing the target level of leverage \( \phi^*_i \). It can also be seen as a scaling factor of \( i \)’s trading size. The parameter \( \kappa_i := b_i/(b_i + h_i) \in [0, 1] \) is the ratio of external funds to total debts, and \( \alpha_{il} := Q_{il}/(\sum_l Q_{il} + \sum_j W_{ij}h_j) \in (0, 1] \) is the ratio of the external asset \( l \) to total assets.

2.5. Leverage-price cycle and market liquidity

In this section, we formalize the leverage-price cycle and link it to the notion of market liquidity. Inspired by the empirical research on the leverage-asset price cycle (Adrian and Shin 2008b, 2010), we now model how the accounting rule in equation (12) impacts the price of (non-paying-dividend) external assets whose dynamics are driven by a standard GBM§

\[
\frac{dS_l}{S_l} = \mu_l dt + \sigma_l dB_l, \quad \forall l \in \Omega_M. \tag{13}
\]

The (bid-ask) trading sizes serve as measures of the strength of the (demand-supply) pressures on external assets at a given time. A large bid size indicates a strong demand for the external

---

†Some candidate explanations in support of the fact that equity remains ‘sticky’ are explained in Adrian and Shin (2011a).

‡Because our aim is to model the leverage-price cycle, this hypothesis allows us to better capture the spillover effect on the real economy that results from the re-adjustments of banks’ claims against real-economy-related activities.

§Where \( B_t \sim N(0, dt) \) is a standard Brownian motion defined on a complete filtered probability space \( (\Omega, \mathcal{F}; \{\mathcal{F}_t\}; \mathbb{P}) \) where \( \mathcal{F}_t = \sigma\{B(s) : s \leq t\} \), \( \mu_l \) is the instantaneous risk-adjusted expected growth rate, \( \sigma_l > 0 \) is the volatility of the growth rate and \( d(B_l, B_k) = \rho_{lk} \).
The parameter $\gamma$ of the dynamic balance-sheet management on the asset price trading volume. More specifically, when prices decrease (14), we can rewrite the price dynamics as follows:

$$\frac{\Delta p}{s_l} = \gamma_l \left( \frac{\Delta Q_l}{Q_l} \right).$$

The parameter $\gamma_l \geq 0$ captures the market impact, i.e. the average price response to trade size. This concept is closely related to the demand elasticity of price and is typically measured as the price return following a transaction of a given volume. In other words, it is the effect that market participants have on price when they buy or sell an external asset. It is the extent to which buying or selling moves the price against the buyer or seller, i.e. upward during buying and downward during selling. Market liquidity (as we will refer to it here) measures the size of the price response to trades and is inversely proportional to the scale of the market impact: $1/\gamma_l$. If trading a given quantity only produces a small price change, the market is said to be liquid. If the trade produces a substantial price change, the market is said to be illiquid. Combining equations (13) and (14), we can rewrite the price dynamics as follows:

$$\frac{\Delta s_l}{s_l} = \gamma_l \left( \frac{\Delta Q_l}{Q_l} \right) dt + \sigma_l dB_l, \quad \forall l \in \Omega_M.$$

Notice that, in principle, changes in price depend on the relative changes in quantity on the banks’ balance sheet, provided that total banks holding of the asset are non-negligible with respect to total holdings of that asset in the economy. In this sense, the market impact parameter $\gamma$ can be thought here to capture both the effect of asset market liquidity and the effect of the relative volume of banks’ holdings with respect to total holdings in the economy.

Equation (15), together with equations (11)–(12), represents the leverage-price cycle. We can describe this positive feedback loop between leverage and prices as follows. At the beginning, the leverage $\phi_l$ is set equal to the target $\phi^*_l$ at a given ‘equilibrium’ price $s^*_l$ (see equation (10)). Therefore, any deviation of $s_l$ from $s^*_l$ leads to a deviation of $\phi_l$ from $\phi^*_l$, which indirectly results from the deviation of $e_l$ from $e^*_l$. This event triggers the previously described accounting rule. When $e_l > e^*_l (e_l < e^*_l)$, the bank has a ‘surplus’ (‘deficit’) of capital to allocate to external assets. More specifically, when prices rise (i.e. $s_l > s^*_l$), the upward adjustment in leverage entails the purchase of external assets. The greater (aggregate) demand for external assets tends to put upward pressure on prices, and the equilibrium price increases. In a downturn, this mechanism is reversed. In a downtrend market (i.e. $s_l < s^*_l$), the downward adjustment of leverage to the target level entails (forced) sales of external assets. The lower (aggregate) demand for external assets puts downward pressure on prices, and the equilibrium price decreases.

### 2.6. The financial system in a nutshell

In summary, the dynamics of the financial market can be described by the following $3 \times n + m$ system of coupled equations with indices range as follows $l = 1, \ldots, m$; $i = 1, \ldots, n$ and parameter values reported in table 1.

$$\begin{align*}
\frac{\Delta s_i}{s_i} &= \gamma_i \left( \frac{\Delta Q_i}{Q_i} \right) dt + \sigma_i dB_i,
\frac{\Delta Q_i}{Q_i} &= \left( \frac{\Delta s_i}{s_i} \right) \left( e^*_i - \phi_i \right) dt,
\frac{\Delta b_i}{b_i} &= \left( \frac{\Delta s_i}{s_i} \right) \left( \frac{\phi_i - e^*_i}{\epsilon_i} \right) dt,
\phi_i &= (h_i + b_i)/ \left( \sum_l Q_{li} s_l + \sum_j W_{ij} \frac{h_j}{1 + \tau_j + \rho_j} \right).
\end{align*}$$

In this scheme, procyclicality is caused by a chain reaction triggered by an exogenous shock (for example, a fall in house prices) and amplified by the interplay between the shock and asset market dynamics. The propagating factor is leverage: when banks are highly leveraged, the initial shock and the ensuing reduction in asset prices induce massive asset liquidation, accentuating the price fall and possibly starting a vicious circle.

For this chain reaction to be fuelled two factors are crucial. Notice that in the system of equation (16), since $\frac{\Delta Q_l}{Q_l}$ depends on $\epsilon$ and $\frac{\Delta s_l}{s_l}$ depends on $\gamma \times \sum_l \frac{\Delta Q_l}{Q_l}$, it follows that the dynamics of $\phi$ depends multiplicatively on the two parameters, e.g. on the product of $\gamma$ and $\epsilon$. Therefore, we can expect that procyclical effects to be strong if both $\gamma$ and $\epsilon$ are large, and to be weak as long as one of the two parameters is small enough. Indeed, for instance, a very small value of $\gamma$ implies that asset sales have no impact on the price and the target leverage in the next time step is not affected, i.e. the procyclicality is weak. This mathematical relation suggest that procyclicality can be tamed by acting either on the tightness of capital requirements or on the market liquidity (which is related to market impact), or both. We will discuss this issue more in detail in the result section.

### 3. Analysis

The system of equation (16) represents a general framework based on which a number of exercises can be conducted and several issues can be investigated. In the present paper, we focus on a specific question that has recently risen to the top of the policy agenda:

**RQ:** How do bank compliance to capital requirements and asset-market liquidity affect the systemic risk of a financial network in the presence of a price shock?

†For the sake of simplicity, in our model, $\gamma_l$ does not change with the trading volume.

‡This ‘flight to quality’ process leads to a price decrease, which in turn forces further selling and further price decreases.
Market procyclicality and systemic risk

The theoretical literature shows that the exact propagation of shocks within the interbank market depends on the specific architecture of bank-to-bank financial linkages (see e.g. Freixas et al. 2000, Allen and Gale 2001). However, the range of possible different architectures shrinks as network density increases. Therefore, when the network density is relatively high, the probability of systemic breakdown depends only weakly on the specific network architecture. Indeed, network density has been found to be a major driver of systemic risk (see e.g. Battiston et al. 2012a, 2012b, Tasca and Battiston 2014). There is also a body of empirical evidence showing that financial networks typically display a core-periphery structure with a dense core and a sparsely connected periphery (see e.g. Iori et al. 2006, Cont et al. 2011, Vitali et al. 2011, Battiston, Puliga et al. 2012). Further, it has been argued that the financial sector has undergone increasing levels of homogeneity because of the spread of similar risk management models (e.g. VaR) and investment strategies—[..] The level playing field resulted in everyone playing the same game at the same time, often with the same ball, Haldane (2009). As a result, one might reason that, banks’ balance sheet structures and portfolios tend to look alike. In particular, in this scenario, banks’ leverage values may differ across banks and over time, but they will remain close to the average value.

Finally, it has been argued that since a simultaneous default implies also that banks are altogether too-big-to-fail, a collective moral hazard emerges (Acharya 2009, Farhi and Tirole 2012), such that banks in the core could be strategically seeking high correlation and interconnectivity with the other banks in the core.

To summarize, the banking system typically displays a core-periphery structure (at least at the country level), where the core is very densely interconnected and not too diverse in terms of business and risk models. This implies that the defaults of the banks are highly correlated and the potential collective moral hazard can reinforce such correlation, implying that banks in the core, in case they default, will do so simultaneously. Empirically, simultaneous defaults are rarely observed because the regulator is forced to step in before they occur, but this is precisely the reason why from a macroprudential perspective it is crucial to study the process that could potentially lead to the default of the whole system.

In this paper, we aim to use the model introduced so far to better understand how banks’ compliance ε to capital requirements interplays with asset-market liquidity γ to give rise to systemic risk. A detailed study of the exit times of each banks in the space of the parameters ε and γ for various scenarios and structures of banks’ portfolios poses non-trivial computational issues. However, in the light of the above considerations, we can reasonably restrict the investigation to the case of a densely connected core of banks. To this end, it makes sense to conduct a mean-field analysis of the model, which as explained below, neglects heterogeneity by captures the average effect of interaction.

More precisely, the scenario we have in mind here intends to give insights on situations like the one of 2007–2009, in which a densely connected core banks were heavily exposed to mortgage-backed securities whose prices were strongly correlated. In order to capture such a scenario, we introduce two main simplifications.

(i) Assumption 1: Mean-field Approximation.

In a dynamical system consisting of n interacting units, taking a ‘mean-field’ approximation means that we replace a variable associated with one unit with its average across the units. This operation makes the variables homogeneous across units but it does not remove the interaction among the variables and hence it preserves the systemic effects arising from the interaction. In other words, a unit interacting with the mean of the population is not the same thing as a unit not interacting at all. In this line of thought, in the following, we want to assume that we can approximate the leverage value of individual banks with the average of the leverage values across all the banks:

\[ \phi_i(t) \simeq \frac{1}{n} \sum_j \phi_j(t) := \phi(t) \quad \forall i \in \Omega_n. \tag{17} \]

This is what we will refer to as mean field approximation of banks’ leverage. We will validate this assumption with some tests in appendix 3. This assumption is justified if balance sheets are sufficiently homogeneous across banks and banks have the same leverage target (i.e. \( \phi^*_i = \phi^* \forall i \)) and the same promptness to capital adequacy (i.e. \( \epsilon_i = \epsilon \forall i \)).

(ii) Assumption 2: Identical external assets.

In order to model the fact that the external assets of banks are driven by a common factor, we assume they all follow a single and the same log normal process of the form:

\[ ds/s = \mu dt + \sigma dB. \]

\(^\dagger\)For an analysis of the case of overlapping portfolios of multiple assets see Tasca et al. (2014), Tasca (2013).

---

Table 1. Range of values for each of the parameters in the model. For the sake of simplicity, we omitted the indices of banks and assets.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Set of values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi^* )</td>
<td>Target leverage</td>
<td>{ ( \phi^* \in \mathbb{R} : 0 &lt; \phi^* &lt; 1 } }</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Variance of external assets</td>
<td>{ ( \sigma \in \mathbb{R} : 0 &lt; \sigma &lt; 1 } }</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Bank reaction to the accounting rule</td>
<td>{ ( \varepsilon \in \mathbb{R} : 0 \leq \varepsilon \leq 1 } }</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Price response of assets to banks’ trades</td>
<td>{ ( \gamma \in \mathbb{R} : \gamma &gt; 0 } }</td>
</tr>
<tr>
<td>( h )</td>
<td>Face value of interbank obligations</td>
<td>{ ( h \in \mathbb{R} : h &gt; 0 } }</td>
</tr>
<tr>
<td>( b )</td>
<td>Face value of external funds</td>
<td>{ ( b \in \mathbb{R} : b &gt; 0 } }</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Sensitivity of interbank obligations to leverage</td>
<td>{ ( \beta \in \mathbb{R} : 0 &lt; \beta &lt; 1 } }</td>
</tr>
<tr>
<td>( r_f )</td>
<td>Risk-free rate</td>
<td>{ ( r_f \in \mathbb{R} : r_f \geq 0 } }</td>
</tr>
</tbody>
</table>
Under the assumptions above, the expression of leverage in (16) simplifies to $\phi = (h + b) / (Q s + h [1 + r_f + \beta \phi]^2)$, which is a quadratic expression in $\phi$. Without loss of generality, we set the risk-free rate equal to zero (i.e., $r_f = 0$). Then, solving for $\phi$ and taking the positive root, we obtain a closed form expression for $\phi$. Overall, equation (16) simplifies to the following system of four coupled equations that will be studied in the remainder of our analysis.

$$\begin{align*}
\frac{dQ}{d\tau} &= \gamma \left( \frac{d\phi}{d\tau} \right) d\tau + \sigma dB \\
\frac{d\phi}{d\tau} &= \left( \frac{\varepsilon}{\varepsilon + \phi} \right) \left( \frac{\phi^* - \phi}{1 - \phi^*} \right) d\tau \\
\frac{d\phi^*}{d\tau} &= \left( \frac{\varepsilon}{\varepsilon + \phi^*} \right) \left( \frac{\phi^* - \phi}{1 - \phi^*} \right) d\tau \\
\phi &= \frac{h(1 - 1 + \beta b - Q s + (4\beta(b + h) Q s + (h - \beta(b + h) + Q s)^2)^{1/2}}{2\beta Q s} \\
\end{align*}$$

The exit time of the variable $\phi$ represents now the simultaneous default of all banks. Notice that in the system of equations above, as in equation (16), there is a cyclical dependence among the variables: $\phi = \phi(s); s = s(Q); Q = Q(\phi)$. The dependence among leverage levels across banks in equations (16) translates now in equations (18) into a dependence of the average leverage on itself at previous times, via the dynamics of prices and balance sheets.

Therefore, although the current formulation neglects the heterogeneity of balance sheets and strategies across banks, it captures (1) the average effect of the interaction of leverage on itself and (2) the interaction between changes in asset prices and changes in leverage.

The interest of the mean field approximation is that it allows to carry out a much more thorough investigation of the space of parameters under many scenarios.

### 3.1 Simulation framework

The simulation procedure is described in detail in appendix 2, while in appendix 3 we compare in a set of simulations the dynamics of the system of equations (16) (with $n = 20$) with its mean field approximation, system of equations (18), along the same paths of idiosyncratic shocks. We find that the difference both in terms of trajectories of leverage and in terms of exit time is small enough to justify the mean field approximation for a systematic investigation.

Because, even in the case of the mean field approximation, there are many possible parameters to vary, we carefully design the simulation framework with respect to the following dimensions: (1) the price shock, (2) the range of $\gamma$ and $\varepsilon$ and (3) the capital structure.

**Aggregate shock** To assess the level of balance-sheet amplification, the exogenous aggregate price shock is assumed to have the following properties: (i) it is common, i.e., the shock is not asset-specific and cannot be diversified away; (ii) it is a single shock, i.e., it hits the external assets only once at the beginning of the simulation process; and (iii) it is uniform, i.e., the strength of the shock is homogeneous across assets.

**Range of $\gamma$ and $\varepsilon$** It is convenient to generate a table $P_{(\varepsilon \times \gamma)}$ that represents the level of market procyclicality as a function of bank compliance with capital requirements $\varepsilon$ and asset-market impact $\gamma$. Accordingly, the market can be weakly, moderately or strongly procyclical (see figure 2). For the former parameter, we choose a set of values that span the entire range of variation: $\varepsilon := \{0.1, 0.2, \ldots, 1\}$. Because our preliminary tests indicated that for values of $\gamma$ that are approximately larger than five, the results of the analysis do not change, so we choose the following set of values $\gamma := \{0.1, 0.2, \ldots, 5\}$

**Capital structure** First, note that under the assumption (2) used to derive the accounting rule in equations (11)-(12), $h(t) = h$ for all $t \geq 0$. Hereafter, for the sake of simplicity, we impose the initial condition $b + h = 1$ and consider three different funding policies based on the imbalance towards external funds versus funds from the interbank market: high imbalance ($b = 0.9 > h = 0.1$), medium imbalance ($b = 0.5 \equiv h = 0.5$), low imbalance ($b = 0.1 < h = 0.9$). We also consider three different levels of target leverage: low target ($\phi^* = 0.7$), medium target ($\phi^* = 0.8$), high target ($\phi^* = 0.9$). As a result, we have nine different capital structures, as illustrated in table 2. The remaining accounting entries for the balance sheets at time $t = 0$ are obtained by solving the following system of equations:

$$\begin{align*}
p + h = h + b + e & \quad \text{(balance-sheet identity)} \\
h = h/(1 + \beta \phi^*) & \quad \text{(market value of interbank claims)} \\
\end{align*}$$

with $p = Qs$. As $\beta \in (0, 1)$, we use $\beta = 0.5$ to represent a scenario in which interest rates on interbank claims have an intermediate level of sensitivity to the obligors’ credit worthiness.
In the table, we focus on an estimate of the probability of default per unit of time. It is thus a useful notion from the point of view of a regulator who has imperfect knowledge of the balance sheets. Therefore, in order to restart the process from the same initial condition, the leverage of the bank with an upper barrier at the value of \( \phi \) is based on Monte Carlo simulations of the system equation (18).\(^\dagger\) The mean time to default is obtained for all of the pairs \( (\varepsilon, \gamma) \) of market procyclicality and systemic risk in figures 4–6 varies as follows.

\[ P = \frac{1}{\bar{T}(\phi_0)}, \]  

based on Monte Carlo simulations of the system equation (18).\(^\dagger\)

We typically run 2000 simulations for each combination of parameter values. The maximum duration of each simulation is \( T = 50,000 \) steps and the step size is \( \Delta t = 0.01 \). More details on the simulation procedure are provided in appendix 2.

The mean time to default is obtained for all of the pairs \( (\varepsilon, \gamma) \) in the table \( P_{(\varepsilon, \gamma)} \) and for all nine capital structures in table 2.

The leverage-price cycle, summarized by the flow-chart in figure 3, is the central part of our Monte Carlo analysis. In brief, at the beginning \( (t = 0) \), the leverage \( \phi \) of the banks is set equal to the target level \( \phi^* \). To this value corresponds the ‘equilibrium’ price value \( s^* := \left[ b + h + \phi^*(b\beta - h + \beta h)\right]/\left[\phi^*Q(1 + \beta\phi^*)\right] \) as derived from equation (16). The initial price shock (−10%) moves the banks out of ‘equilibrium’ by deviating \( s \) from \( s^* \) and \( \phi \) from \( \phi^* \). To drive the leverage back towards \( \phi^* \), banks adjust their external asset holdings and external funds according to the accounting rule, i.e. the combination of the second and third expression in equation (18). The re-sizing of banks’ balance sheets has a market impact on asset prices which in turn further influence the balance sheet entries. This effect completes the leverage-asset price cycle (see figure 3).

### 3.2. Monte Carlo simulations

Consider to run many times a stochastic process describing the leverage of the bank with an upper barrier at the value of one and to restart the process from the same initial condition \( \phi_0 < 1 \) every time it exits the barrier. Thus, the longer it is the average exit time, the smaller it is the probability per unit of time that the bank exits the barrier. If we now select a given point in time and we do not know exactly where the process is at the moment, then the inverse of the average exit time gives us an estimate of the probability of default per unit of time. It is thus a useful notion from the point of view of a regulator who has imperfect knowledge of the balance sheets. Therefore, in the following, we focus on an estimate of the probability of systemic default, measured as the inverse of the mean time to default:

\[ P = \frac{1}{\bar{T}(\phi_0)}, \]  

weak procyclicality—left-lower corner of \( P_{(\varepsilon, \gamma)} \). Systemic risk is at its minimum level. Banks are in the region of weak market procyclicality characterized by a liquid asset market (i.e. \( \gamma \gtrsim 0 \)) and a weak adjustment of any leverage deviation from the target level (i.e. \( \varepsilon \gtrsim 0 \)). Even though the price shock induces some asset liquidations, it is immediately absorbed by the system because of a sluggish balance-sheet management and a liquid asset market. Eventually, systemic risk remains at low levels. Medium procyclicality—left-upper corner of \( P_{(\varepsilon, \gamma)} \). Systemic risk is at a moderate level. On the one side, banks promptly comply with capital requirements (i.e. \( \varepsilon \gtrsim 1 \)) and react to price shocks by selling orders of large size. Therefore, massive liquidation of banks’ assets may put further downward pressure on asset prices. On the other

\[ P = \frac{1}{\bar{T}(\phi_0)}, \]  

4. Results

Figures 4–6 show the probability surface of the systemic default event as a function of (1) the level of market procyclicality and (2) the banks’ capital structure. The capital structure, instead, refers to the balance between external and interbank funding sources. Finally, each figure corresponds to a different level of target leverage: \( \phi^* = 0.7, 0.8, 0.9 \).

### 4.1. Market procyclicality

In connection with the areas of the table \( P_{(\varepsilon, \gamma)} \) of market procyclicality in figure 2, the systemic risk in figures 4–6 varies as follows.

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>Simulation setting</th>
<th>Initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma = 0.1 )</td>
<td>shock = −10%</td>
<td>( b_0 = 0.1, 0.5, 0.9 )</td>
</tr>
<tr>
<td>( r_f = 0 )</td>
<td>( N = 2000 )</td>
<td>( Q_0 = 1 )</td>
</tr>
<tr>
<td>( \beta = 0.5 )</td>
<td>( \Delta t = 0.01 )</td>
<td>( s_0 = s^* \times \text{(shock + 1)} )</td>
</tr>
<tr>
<td>( \varepsilon = 0.1 : 0.1 : 1 )</td>
<td>( T = 50,000 )</td>
<td>( \gamma = 0.1 : 0.1 : 5 )</td>
</tr>
<tr>
<td>( h = 0.1, 0.5, 0.9 )</td>
<td>( \phi = 0.7, 0.8, 0.9 )</td>
<td>( \phi^* = 0.7, 0.8, 0.9 )</td>
</tr>
</tbody>
</table>

\[ \dagger \] The values of the parameters used in the simulations are reported in table 3, and the procedure is described in greater detail in appendix 2.

### Table 2

<table>
<thead>
<tr>
<th>( \phi^* = 0.7 )</th>
<th>( \phi^* = 0.8 )</th>
<th>( \phi^* = 0.9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
<td>Assets</td>
</tr>
<tr>
<td>( b &gt; h )</td>
<td>( h = 0.074 )</td>
<td>( h = 0.1 )</td>
</tr>
<tr>
<td>( p = 1.354 )</td>
<td>( b = 0.9 )</td>
<td>( e = 0.25 )</td>
</tr>
<tr>
<td>( b = h )</td>
<td>( h = 0.37 )</td>
<td>( h = 0.5 )</td>
</tr>
<tr>
<td>( p = 1.058 )</td>
<td>( b = 0.5 )</td>
<td>( p = 0.892 )</td>
</tr>
<tr>
<td>( b &lt; h )</td>
<td>( h = 0.66 )</td>
<td>( h = 0.9 )</td>
</tr>
<tr>
<td>( p = 0.768 )</td>
<td>( b = 0.1 )</td>
<td>( p = 0.607 )</td>
</tr>
<tr>
<td>( e = 0.428 )</td>
<td>( e = 0.25 )</td>
<td>( e = 0.25 )</td>
</tr>
</tbody>
</table>

### Table 3

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>Simulation setting</th>
<th>Initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma = 0.1 )</td>
<td>shock = −10%</td>
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</tr>
<tr>
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</tr>
<tr>
<td>( \beta = 0.5 )</td>
<td>( \Delta t = 0.01 )</td>
<td>( s_0 = s^* \times \text{(shock + 1)} )</td>
</tr>
<tr>
<td>( \varepsilon = 0.1 : 0.1 : 1 )</td>
<td>( T = 50,000 )</td>
<td>( \gamma = 0.1 : 0.1 : 5 )</td>
</tr>
<tr>
<td>( h = 0.1, 0.5, 0.9 )</td>
<td>( \phi = 0.7, 0.8, 0.9 )</td>
<td>( \phi^* = 0.7, 0.8, 0.9 )</td>
</tr>
</tbody>
</table>
side, the asset market is very liquid (i.e. \( \gamma \geq 0 \)). Therefore, potentially large quantities of the assets can be sold or bought without significant impact on their market price. Systemic risk remains at medium levels until the liquidity does not ‘evaporate’ from the market.

**Medium procyclicality**—right-lower corner of \( P(\epsilon \times \gamma) \). Systemic risk is at a moderate level. The asset-market is illiquid (i.e. \( \gamma \leq 5 \)), and banks loosely comply with capital requirements (i.e. \( \epsilon \geq 0 \)). In this situation, the selling orders executed by the banks in response to the price shock are limited in size. Combined with a liquidity shock, these trades produce only a small price change, which is sufficient, however, to trigger a weak spiral of asset-price devaluation. This spiral further degenerates the banks’ balance sheets and deteriorates the leverage. As a consequence, systemic risk slightly increases.

**Strong procyclicality**—right-upper corner of \( P(\epsilon \times \gamma) \). Systemic risk is at its maximum level. Banks are in the region of strong market procyclicality characterized by the extremely dangerous combination of a highly illiquid asset market (i.e. \( \gamma \leq 5 \)) and prompt responses to any deviation of the leverage from the target level (i.e. \( \epsilon \leq 1 \)). In this context, banks react to the price shock via selling orders of a significant size. If at the same time a liquidity shock materializes such that buyers temporarily retreat from the market, external assets can only be sold at ‘firesale prices’. Eventually, the market crash degrades the balance sheets and both leverage and the default risk increase.

### 4.2. Capital structure

Banks’ capital structures also play an important role in systemic risk except under weak market procyclicality. Indeed, when banks’ exposure to external funds is greater (i.e. \( b > h \)), a price shock will have a greater effect on banks’ net worth, and systemic risk will be higher (as indicated by the light-grey surfaces in figures 4–6). In contrast, when banks’ positions are more tilted towards the interbank market, the price shock is damped down because it mainly propagates via a drop in the market value of the interbank claims (as indicated by the dark-grey surfaces in figures 4–6). In this case, the systemic risk is lower.

This result is in line with previous works (Nier et al. 2007, Montagna and Lux 2014) and it arises because we are looking at a relative balance sheet exposure to external assets. However, as one would expect, everything else the same, a higher absolute exposure of banks to the interbank market increases their leverage and thus the systemic risk.

Interestingly, the gaps between the levels of systemic risk associated with the three different funding policies are amplified at high levels of market procyclicality (see the border between the surfaces in figures 4–6). Finally, the higher the target leverage level is, the more pronounced the effects (compare figures 4–6). Indeed, leverage is the propagating factor of a price shock: if banks did not target leverage, letting equity absorb the shock, the vicious circle would be mitigated or even eliminated.

Overall, the results suggest that to mitigate the systemic effects of an unexpected price shock, two complementary policy rules can be implemented: (1) one can allow banks to weaken their compliance in adjusting their leverage to meet the target level (i.e. \( \epsilon \) will decrease towards zero);† or (2) one can set

†Based on our Monte Carlo trials, we find that the leverage follows a slower mean reversion process towards the target level. Interestingly, this effect could explain the empirical puzzle recently discovered by Adrian and Shin (2008b) concerning the slow mean reversion over time exhibited by the (reported) leverage ratios.
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Figure 4. Surface probability of systemic default when banks’ target leverage is 0.7. In the x-axis, \( \gamma \) represents the average price response to trades. In the y-axis, \( \varepsilon \) represents the intensity of bank compliance with capital requirements. The light grey surface shows the probability of systemic default when banks have high imbalance towards external funds \((b > h)\). The grey surface shows the probability of systemic default when banks have medium imbalance towards external funds \((b = h)\). The dark grey surface shows the probability of systemic default when banks have high imbalance towards external funds \((b > h)\).

Figure 5. Surface probability of systemic default when banks’ target leverage is 0.8. For a description of the surfaces see figure 4.

Figure 6. Surface probability of systemic default when banks’ target leverage is 0.9. For a description of the surfaces see figure 4.

Indeed, for containing macro-prudential risks, leverage caps could be helpful if other solutions based on capital or contingent rules turn out to be too costly or difficult to implement. However, one drawback of this rule lies in its difficulty to be harmonized among different business models and cross-border banks. Moreover, leverage caps should be robust to: (i) the accounting treatment of off-balance operations (incl., derivatives and hedges); and (ii) the treatment of leverage embedded in structured finance products.

5. Concluding remarks

In this paper, we combined a balance sheet approach with a dynamic stochastic setting to investigate the impact of market procyclicality on systemic risk. One of the novelties of our work is that it provides a quantitative representation of the notion of market procyclicality using a table whose dimensions correspond to (1) the level of bank compliance with capital requirements (controlled by the parameter \( \varepsilon \)) and (2) the degree of asset market liquidity (controlled by the parameter \( \gamma \)).

The general approach is to perturb the system with an aggregate price shock and analyse the probability of systemic default in critical regions of the table of market procyclicality. In our model, systemic defaults are a result of drops in asset prices, which are endogenously driven by bank trades. Thus, the leverage-asset price cycle is characterized by a ‘self-fulfilling’ dynamics. Essentially, over-leveraged banks (which are more likely to default) tend to liquidate their assets after negative price variations. However, their market impact further decreases asset prices. Hence, at the heart of our model are pecuniary externalities in the form of direct asset price contagion (via overlapping portfolios) and indirect asset price contagion (via interbank claims). In particular, we stress the policy implications of the interplay between bank capital requirements and asset market liquidity. When the asset market

†The G20 group of leading countries agreed on September 2012 to introduce a leverage ratio on banks by the end of 2012, as part of the Basel III reform from the global Basel Committee on Banking Supervision, to make the sector less risky.
is very liquid, even a strong compliance with capital requirements, which are usually alleged to be procyclical, do not in fact increase the probability of systemic default. Conversely, when the asset market is very illiquid, even a weak compliance with capital requirements increases the probability of systemic default.

Overall, this paper sheds light on the tension between (1) the individual incentive to aim for a particular ratio of VaR to economic capital and reduce idiosyncratic shocks through asset diversification and (2) the homogenized system that results from the fact that banks adopt the same business and risk management practices. From an individual firm perspective, the application of the accounting rule and asset diversification is a sensible strategy. It is important to note that every bank is acting perfectly rationally from its own individual perspective. However, from a systemic perspective, this strategy generates an undesirable result. In fact, when banks start moving in a ‘synchronized’ manner, they may amplify the effects of even small external shocks on assets dispersed across banks.

One possible illustrative application of our modelling framework is the US sub-prime crisis (2007–2009). In that crisis, the external asset market would be the market for mortgage-backed securities, which was hit by a fundamental shock (i.e. the collapse of the US housing bubble). As in our model, many of the largest financial institutions exposed to those assets were highly target-leveraged, were interconnected via the interbank market and made use of mark-to-market accounting. At the peak of the crisis, banks stopped lending to each other, and the market froze. To reduce the risk of systemic collapse, the Federal Reserve injected additional liquidity and helped banks to reduce their exposure to the external asset market by buying a portion of the mortgage-backed securities. Therefore, our analysis suggests that policy-makers should employ macro-prudential supervisory risk assessment policies in coordination with monetary policies to compensate for the effect of market-wide liquidity in the presence of aggregate shocks.

The simplicity of our theoretical framework allows for the present research to be extended in several ways. For instance, the core of banks could be understood as ‘too-big-to-fail’, and one could try to model the moral hazard problem,† which could induce excessive risk-taking with regard to external assets. Researchers might also consider the effect of the heterogeneity of banks’ balance-sheet structures and target leverage levels as well as heterogeneity in terms of asset price volatility and drift. Finally, one could include dynamic link formation both in the interbank network and in the network of banks and external assets. Finally, it would be possible to use different formulations of the asset price dynamics for instance by introduce a time-dependent drift and variance in order to capture an evolving economic environment.

Funding

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References


†Moral hazard would arise from banks’ expectations concerning the intervention of the central bank in providing liquidity or reducing exposure to external assets.
Appendix 1. An accounting rule based on target leverage

In this section, we derive the accounting rule in equations (11)–(12), according to which a generic bank \( i \) in \( \Omega_n \) adjusts its balance sheet entries in response to an asset price change.

For convenience, let us consider a process in discrete time, first. At time \( t = 0 \), bank \( i \) starts its activity with a target leverage, \( \phi_i(0) = \tilde{\phi}_i^* \). As pointed out in section 2.4, the accounting rule is based on the underlying assumption that bank \( i \) keeps its debt/credit positions fixed in the interbank market: the bank only adjusts the quantity of external assets and external funds: \( h_i(t) = h_i(0) = \sum_{j \neq i} W_{ij} h_j(0) \) for all \( t \geq 0 \) and all \( i, j \in \Omega_n \). Thus, the initial leverage can be written as \( \phi_i(0) = \phi_{i}^* = \frac{h_{i}(t)+\phi_{i}(0)}{\phi_{i}(1)} \).

At time \( t = 1 \), for the sake of simplicity, let only one external asset \( l \in \Omega_n \) be shocked. \textit{Ceteris paribus} (i.e. \( h_i(t) \) and \( Q_{il}(t) \) remain as they were at time \( t = 0 \)), the new price value \( \delta(1) \) implies the following leverage:

\[
\phi_i(1) = \frac{\sum_{j \neq i} Q_{il}(0) h_j(1) + \sum_{j \neq i} W_{ij} h_j(0) \neq \phi_{i}^*}.  
\]

After the price shock, at time \( t = 2 \), the bank is able to adjust its leverage to the target level by changing the quantity of external asset \( l \) held on the asset side (i.e. \( \sum_{j \neq i} Q_{il}(t) \rightarrow \sum_{j \neq i} Q_{il}(2) \)), and of the external debts on the liability side (i.e. \( b_{i}(t) \rightarrow b_{i}(2) \)). As a result, the new leverage at time \( t = 2 \) is equal to the target level, i.e. \( \phi_i(2) \equiv \tilde{\phi}_i^* \). Expanding the equivalence \( \phi_i(2) \equiv \phi_i(2) \equiv \tilde{\phi}_i^* \) yields

\[
\phi_i(2) = \frac{h_i + b_i(2)}{\sum_{j \neq i} Q_{il}(2) s_j(1) + \sum_{j \neq i} W_{ij} h_j(1)} = \frac{h_i + b_i(0)}{\sum_{j \neq i} Q_{il}(0) s_j(1) + \sum_{j \neq i} W_{ij} h_j(1)} \equiv \tilde{\phi}_i^*.  
\]

Now, the equation for leverage at time \( t = 1 \) can be re-written as

\[
\phi_i(1) = \frac{h_i + b_i(2)}{\sum_{j \neq i} Q_{il}(2) s_j(1) + \sum_{j \neq i} W_{ij} h_j(1) + \sum_{j \neq i} W_{ij} h_j(0) \neq \phi_{i}^*}.  
\]

where \( \Delta Q_{il} \) indicates the variation in the quantity of external assets in bank \( i \)’s portfolio due to a change in its holdings of external asset \( l \). Equation (A2) can be re-written as

\[
\frac{h_i + b_i(0)}{\phi_{i}(1)} + \Delta Q_{il}(2) s_j(1) = \frac{h_i + b_i(2)}{\tilde{\phi}_i^*}.  
\]

Based on the assumptions associated with the accounting rule as indicated in section 2.4, the following identity holds true:

\[
b_i(2) = b_i(0) + \Delta Q_{il}(2) s_j(1).  
\]

Then, substituting (A4) into (A3) yields

\[
\Delta Q_{il}(2) s_j(1) = \frac{h_i + b_i(0) + \Delta Q_{il}(2) s_j(1)}{\phi_{i}^*} \neq \Delta Q_{il}(2) s_j(1) \neq \frac{h_i + b_i(0) + \Delta Q_{il}(2) s_j(1)}{\phi_{i}^*},  
\]

from which, with little rearrangements we obtain

\[
\Delta Q_{il}(2) s_j(1) = \frac{h_i + b_i(0)}{\phi_{i}(1)} \left( \phi_{i}^* - \phi_{i}(1) \right)  
\]

Therefore, the rate of change in the demand for asset \( l \) from bank \( i \) is:
Appendix 2. Numerical analysis

The boundary conditions are: (i) \( \frac{d}{dt} \theta^* - \frac{d}{dt} \theta_t = 0 \) \( \forall t \geq 0 \) and \( \theta_t \) is the value of the external asset \( l \) to the total asset value held by the bank.

The relative change in debt is easily obtained from equation (A4). However, in the presence of market frictions, the bank may react with a certain (in)elasticity to deviations in leverage \( \phi_t \) from the target level. Therefore, in the following, we assume that the relative change in debt is further multiplied by the factor \( \varepsilon_l \in (0, 1] \), which captures the speed at which the bank adjusts the balance sheet to the target. When \( \varepsilon_l = 1 \), the balance sheet adjustment would allow the bank to reach the target leverage in only one step, neglecting the effect of asset sales/purchases on the market price.

\[
\frac{b_{l}(t) - b_{l}(0)}{b_{l}(0)} = \varepsilon_l \frac{\phi_{t}^* - \phi_{t}}{1 - \phi_{t}^*} = \varepsilon_l \frac{\phi_{t}^* - \phi_{t}}{1 - \phi_{t}^*}, \quad (A7)
\]

where \( \varepsilon_l = \frac{b_{l}(t) - b_{l}(0)}{b_{l}(0)} \) is the ratio of external debts to the total debts.

From the equations we can derive the dynamics in continuous time for the demand for external assets and the amount of external debts:

\[
\frac{dQ_t}{Q_t} = \frac{\varepsilon_l}{\alpha_{ij}} \left( \frac{\phi_{t}^* - \phi_{t}}{1 - \phi_{t}^*} \right) dt \quad (A8)
\]

\[
\frac{d\alpha_{ij}}{\alpha_{ij}} = \left( \frac{\phi_{t}^* - \phi_{t}}{1 - \phi_{t}^*} \right) dt. \quad (A9)
\]

The boundary conditions are: (i) \( \frac{dQ_t}{Q_t} = \sum_{l \in \Omega_m} \frac{dQ_t}{Q_t} = 0 \) for some \( l \in \Omega_m \); (ii) \( \frac{dQ_t}{Q_t} \geq -Q_t \) for all \( l \in \Omega_m \) and \( l \in \Omega_m \), and (iii) \( d\alpha_{ij} \geq -\alpha_{ij} \) for all \( l \in \Omega_m \). The accounting rule in equation (A8) can be easily generalized if bank \( i \) changes the quantity of more than one external asset: For example, the rule for changing the quantity of all the assets in \( \Omega_m \) is

\[
\frac{dQ_t}{Q_t} = \left( \frac{\phi_{t}^* - \phi_{t}}{1 - \phi_{t}^*} \right) dt \quad (A10)
\]

where \( Q_t = \sum_{l} Q_{l t} \) and \( \alpha_{ij} = \sum_{l} \alpha_{l j} \).

Appendix 2. Numerical analysis

In this section, we show how to obtain the mean time to default from the system (18), through simulations of the SDEs. The first step is to simulate the standard Brownian motion. Then, we look at stochastic differential equations, after which we compute the exit time for the process \( \phi_{t} \) through the upper barrier fixed at one.

B.2. Simulation of the SDEs

We use the Euler-Maruyama method. First, we discretize the dynamics of the processes \( \{s_{l} \geq 0, \{Q_{l} \geq 0 \} \} \) into the discrete time version of the last expression in equation (18). It is unnecessary to derive the dynamics of \( \phi_{t} \) via Ito’s Lemma from the dynamics of \( \{s_{l} \geq 0, \{Q_{l} \geq 0 \} \} \) and \( b_{l} \). We directly use the last expression in equation (18), which is the closed-form solution for \( \phi_{t} \) in terms of \( s, Q, b_{l} \).

\[
\frac{Q_{l t}(2) - Q_{l t}(0)}{Q_{l t}(0)} = \sum_{l} W_{l} h_{l j} + \frac{Q_{l t}(0) - Q_{l t}(1)}{Q_{l t}(1)} \frac{\phi_{t}^* - \phi_{t}}{1 - \phi_{t}^*}, \quad (A6)
\]

where \( \alpha_{ij} = \frac{Q_{j t}(0) - Q_{j t}(1)}{Q_{j t}(1)} \) is the ratio of the value of the external asset \( l \) to the total asset value held by the bank.

B.3. Estimation of expected time to default for the mean-field approximation

The expected exit time can be estimated in the following manner. We simulate a single sample path for the variable \( \{s_{l} \geq 0, \{Q_{l} \geq 0 \} \} \) in the system of equations in section 3, we carry out a comparison between simulations of the system and its mean field approximation. In order to test the validity of the mean field approximation described in section 3, we carry out a comparison between simulations of the system and its mean field approximation. In order to test the validity of the mean field approximation described in section 3, we carry out a comparison between simulations of the system and its mean field approximation. In order to test the validity of the mean field approximation described in section 3, we carry out a comparison between simulations of the system and its mean field approximation. In order to test the validity of the mean field approximation described in section 3, we carry out a comparison between simulations of the system and its mean field approximation.

### B.3.1. Simulation of the SDEs

We begin by considering a discretized version of Brownian motion. We set the step-size as \( \Delta t \). According to the properties of Brownian motion, we find

\[
B_{t} = B_{t - \Delta t} + dB_{t} \quad \forall t = 1, \ldots, L. \quad (B1)
\]

Here, \( L \) denotes the number of steps that we take with \( t_0, t_1, \ldots, t_L \) as a discretization of the interval \([0, T] \), and \( dB_t \) is a normally distributed random variable with mean zero and variance \( \Delta t \). Expression (B1) can be seen as a numerical recipe to simulate Brownian motion.

### B.3.2. Simulation of the SDEs

We use the Euler-Maruyama method. First, we discretize the dynamics of the processes \( \{s_{l} \geq 0, \{Q_{l} \geq 0 \} \} \) into the discrete time version of the last expression in equation (18). It is unnecessary to derive the dynamics of \( \phi_{t} \) via Ito’s Lemma from the dynamics of \( \{s_{l} \geq 0, \{Q_{l} \geq 0 \} \} \) and \( b_{l} \). We directly use the last expression in equation (18), which is the closed-form solution for \( \phi_{t} \) in terms of \( s, Q, b_{l} \). Therefore, the discrete version of the system of equations in section 2.2 reads as follows:

\[
l_{1} = \frac{Q_{l t}(2) - Q_{l t}(0)}{Q_{l t}(0)} = \sum_{l} W_{l} h_{l j} + \frac{Q_{l t}(0) - Q_{l t}(1)}{Q_{l t}(1)} \frac{\phi_{t}^* - \phi_{t}}{1 - \phi_{t}^*},
\]

\[
Q_{l t}(2) = Q_{l t}(1) + \mu \phi_{t}^*(t - \phi_{t}^*), \quad \gamma, \epsilon, \alpha_{l t} \Delta \tau = (s_{l t} - s_{l t-1}) dB_{l t}.
\]

\[
\frac{Q_{l t}(2) - Q_{l t}(0)}{Q_{l t}(0)} = \sum_{l} W_{l} h_{l j} + \frac{Q_{l t}(0) - Q_{l t}(1)}{Q_{l t}(1)} \frac{\phi_{t}^* - \phi_{t}}{1 - \phi_{t}^*}.
\]

where \( \alpha_{l t} = \mu (s_{l t} - s_{l t-1}) dB_{l t} \). Here, \( s_{l t} \) is the quantity of asset \( l \) at time \( t \). The expected default time \( \tau(\phi_{t}) \) is approximated by the average exit \( \bar{\tau}(\phi_{t}) \) across the sample paths.

\[
\bar{\tau}(\phi_{t}) = \frac{1}{N} \sum_{j=1}^{N} \bar{\tau}(\phi_{t}) = \int_{0}^{T} t f_{g}(1, t \mid \phi_{t}, 0) dt.
\]

where \( f_{g}(1, t \mid \phi_{t}, 0) \) is the probability distribution function of the exit time.

### B.3.3. Comparison of simulations of the \( n \)-bank system and its mean field approximation

In order to test the validity of the mean field approximation described in section 3, we carry out a comparison between simulations of the simultaneous dynamics of \( n \)-bank system and simulations of its mean field approximation. We start with the case in which the network of interbank liabilities is a fully connected graph (i.e. every bank lends to and borrows from all the others). In order to test the robustness of the approximation in the presence of a small heterogeneity across banks, we also consider the case in which:

- the network structure is a random graph with density of links equal to 1/2 (half of the possible links exist);
- the values of the interbank exposures of each bank are
equally weighted towards the counterparties, i.e. equal to $1/k_{\text{out}}$, where $k_{\text{out}}$ is the number of borrowers.

- the exposure to the external assets of each bank are randomly distributed around a mean value $Q_{i} = 1$ with a uniform distribution with maximum variation of $\pm 10\%$.

In the following, we report only the result for this weakly heterogenous case, since the deviations between equations (16) and (18) are smaller in the homogenous case. A part from the specification above, the parameters are set as in section 3.2, with $\phi^* = 0.8$.

We first compare the values of leverage obtained with the two systems of equations (16) and (18) on individual runs for some chosen parameters. In order for the comparison to be meaningful, in each run the sample path of the idiosyncratic shocks $dB$ to the external asset is the same in the two cases. Some examples of sample paths for some chosen runs are reported in figure C1. The grey lines represent the leverage of each bank over time simulated with equations (16) and the blue triangles are the average across the leverage values of the $n$ banks. Moreover, the red stars are the values of mean field leverage simulated with equations (18). The different plots refer to increasing values of market impact $\gamma$, while the capital adequacy promptness $\epsilon$ is set to 0.75. As we can observe, the mean field approximation follows closely the average leverage across banks, with some deviations when the most leveraged banks start to exit the barrier at 1.

In some runs, the trajectories of certain banks appear to be closer to each other than others. This is due to the specific structure of the random graph in which a given bank happens to be exposed to certain banks and not to others. In order to avoid a bias in the results, the random graph of liabilities is assigned anew in every run.

We also compute some statistics of the difference, at each point in time, between the mean field leverage of equation (18) and the average of the leverage values across the $n$ banks. The following quantities are computed from a sample of 1000 runs. Figure C1(d) shows that the median values (blue) of such differences remains around $5 \times 10^{-4}$ across the range of $\gamma$ values. Recall that $\phi$ values are comprise in $[0, 1]$, so the difference is on average smaller than 0.05 %. Moreover, the interquartile range is around $2 \times 10^{-3}$, i.e. $50\%$ of the times the difference between the two curves is smaller than 0.2 %.

Since the quantity we are eventually interested in is the mean exit time, in the case of $n$ banks, in each run we compute the median across the $n$ banks of the first exit time of each bank in the simultaneous $n$-bank dynamics. From a systemic risk perspective, while the first exit time across banks may be subject to larger fluctuations, the median exit time across banks has the meaning of representative exit time in the system and it is a more robust estimate. This is the quantity that we compare to the exit time from the mean-field approximation.

Figure C2 shows the values of exit time of the two systems in 100 runs for two values of market impact $\gamma = 0.75$ (a) and $\gamma = 2$ (b). For better readability, the exit time values of the $n$-bank system (blue) in each run are sorted by ascending order. The grey lines represent the exit times of each individual bank in the simultaneous dynamics of 20-bank system in that run. The red line represent the exit time in the mean field approximation in the same run.

Figure C3(a) shows that both the mean exit time for the $n$-banks system (i.e. the average across the banks in each run, in blue) and for the mean field approximation decrease with $\gamma$ and are close to each other well within the interquartile range (dotted lines). Figure C3(b) shows that the median difference (blue) among the exit time of the two systems remains around 1 across the $\gamma$ range (recall that the exit time is typically larger than 100 in our setting), while the interquartile range (red) decreases with $\gamma$ from about 30 (for small values of $\gamma$, when the exit times is typically 200–800) to about 2 (for larger values of $\gamma$ when the exit time is typically larger than 40. We conclude that the trajectories of the two system are typically very close to each other and that the exit time of the mean field approximation is a good estimate of the exit time of the system of $n$ banks, at least when the interbank market is dense, even in the presence of some heterogeneity.
Figure C1. Sample paths of leverage versus time in a given run for the $n$-banks system (blue) and the mean field approximation (red) from equations (18). Grey lines: the leverage of each bank over time simulated with equations (16). Blue triangles: the average across the leverage values of the $n$ banks. Red: the mean field leverage simulated with equations (18). Unless specified, parameters are set for all plots as in section (3.2). Target leverage $\phi^* = 0.8$, number of banks $n = 25$. The plots (a) (b) (c) refer to increasing values of market impact $\gamma$, while the capital adequacy promptness $\epsilon$ is set to 0.75. (d) Median values (blue) of differences on $\phi$ between the two systems and interquartile range (red).
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Figure C2. Exit time obtained from the $n$-bank system (median across banks in blue, exit times of individual banks in grey) and from the mean-field approximation (in red) from equation (18). y-axis: exit times in a given simulation run. x-axis: the 100 simulation runs are sorted by increasing exit time in the $n$-bank case. Unless specified, parameters are set for all plots as in section 3.2. Target leverage $\phi^* = 0.8$, number of banks $n = 25$. (a) $\gamma = 0.75$; (b) $\gamma = 2$.

Figure C3. Statistics about trajectories and exit time obtained for the $n$-bank system (blue) and the mean field approximation (red) from equation (18). Unless specified, parameters are set for all plots as in section 3.2. Target leverage $\phi^* = 0.8$, number of banks $n = 25$. (a) Median values (blue) of differences on $\phi$ and interquartile range (red). (b) Mean exit time for the $n$-banks system (blue) and for the mean field approximation (red) as a function of $\gamma$. Interquartile ranges are shown as dotted lines. c) Median difference (blue) among the exit time of the two systems and interquartile range (red) as a function of $\gamma$. 